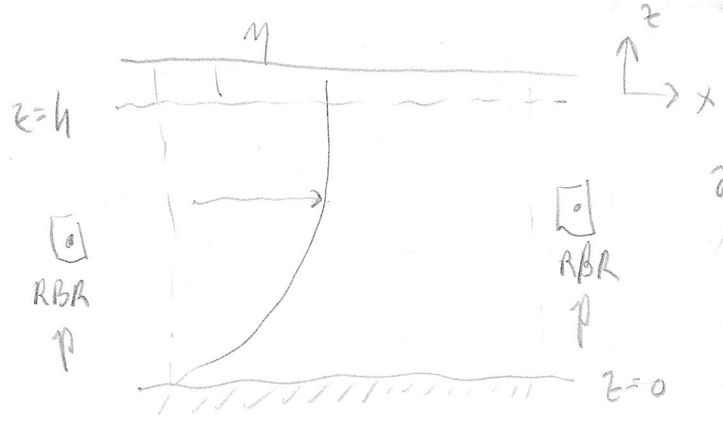
7/23/2019 ①

RG

$\boxed{\text{Unstratified Boundary Layers + Turbulence}}$



Momentum $\boxed{x \text{ mom}}$

$$\cancel{\frac{\partial u}{\partial t}} + \cancel{\underline{u}\cdot\nabla u} - \cancel{fv} = \underbrace{-\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2}}_{\text{known cond.}}$$

$$\overbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial x}}^{\text{neg.}} = \overbrace{\nu}^{\text{neg}}\frac{\partial^2 u}{\partial z^2}$$

$\frac{\partial u}{\partial z} = 0$ at $z = h$

$u = 0$ at $z = 0$

integrate twice

$$u = \frac{-\frac{1}{\rho_0}\frac{\partial p}{\partial x}}{\nu} z\left(h - \tfrac{1}{2}z\right)$$

$$u' = (\quad)(h - z)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x} = -1\times10^{-4}\ \mathrm{m\,s^{-2}} \quad (1\ \mathrm{cm} / 1\ \mathrm{km})$$

$$\nu = 1\times10^{-6}\ \mathrm{m^2/s}$$

$$\Rightarrow \quad u_{surf} = 5{,}000\ \mathrm{m/s}$$

$\therefore$ it is not laminar flow!

Rewrite x-mom using stress

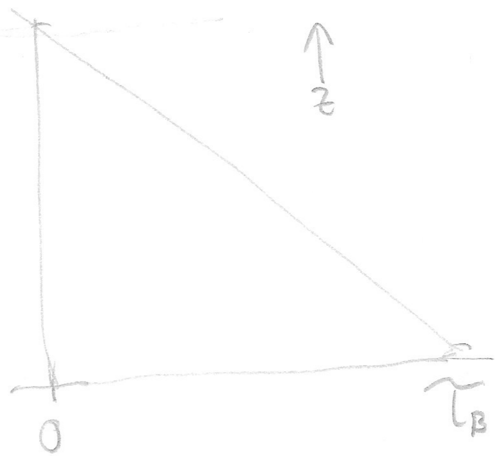
$$0 = \frac{-1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial \tau}{\partial z}$$ τ = shear stress

Laminar Flow $\frac{\tau}{\rho_0} = -\nu \frac{\partial u}{\partial z}$ sign convention ?

Turbulent Flow $\frac{\tau}{\rho_0} = -\langle u' w' \rangle$

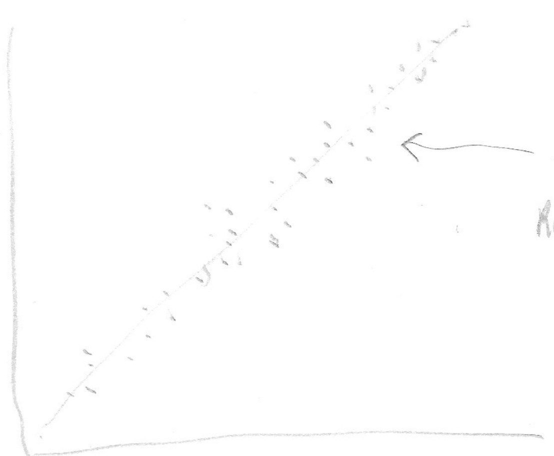
so after vertical integral

$$\overbrace{\frac{-1}{\rho_0}\frac{\partial p}{\partial x}}^{+} = \overbrace{\frac{1}{\rho_0}\frac{\tau_B}{h}}^{+}$$



Empirical Evidence

$\frac{-1}{\rho_0}\frac{\partial p}{\partial x}$ ~ $\frac{\tau_B}{\rho_0 h}$



$\bar{u}^2$

slope = Cd
Range $1.5 - 3.5 \times 10^{-3}$

$$\frac{\tau_B}{\rho} \cong C_d U^2$$ but to account for changing sign

$$\frac{\tau_B}{\rho_0} \cong C_D |\bar{u}| \bar{u}$$

$\sqrt{\bar{u}^2 + \bar{v}^2}$

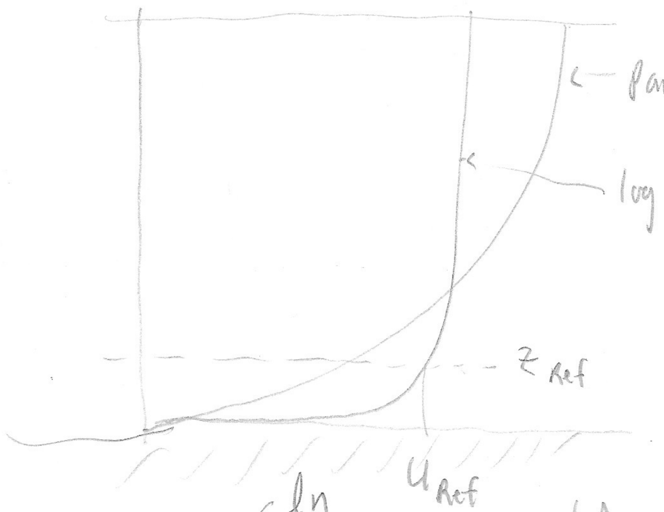
coral reef	$C_D \approx 0.1$
North River Salt Marsh	$C_D \approx 0.02$
Shallow Estuaries	$C_D = 0.003$
Deeper Estuaries	$C_D = 0.002$
Deep Shelf Smooth Bottom	$C_D = 0.001$

Exercise: what is stress + bottom pressure in your field site [Pa]?

$$\tau = \rho C_d U^2 = 10^3 \cdot 3 \times 10^{-3} \cdot 1 = 3 \text{ Pa}$$

$$p = \rho g\, 100 = 10^3 \cdot 10^4 \cdot 10^2 = 10^6 \text{ Pa}$$

The Log Layer



$$u(z) = \frac{u_*}{K} \log\left(\frac{z}{z_0}\right)$$

ln

$$u_* = \left(\tau_* / \rho_0\right)^{1/2}$$

K = "von Karman's const"
= 0.41

z_0 = "z naught"
Roughness length

Nikuradse : sand grain roughness

$z_0 \approx \frac{1}{30} d_{Nikuradse}$

Also works for sand wave ripple height

$\boxed{\text{Relating } u_* \text{ + } C_d}$

$$u_*^2 = C_D \overline{u}^2 = C_{DR} u_{Ref}^2$$

$z_{Ref} = 1$ m

at a reference height like 1 m

$$u(z_{Ref}) = \frac{u_*}{Z} \log\left(\frac{z_{Ref}}{z_0}\right)$$

solve for C_{DR}

$$C_{DR} = \left[\frac{K}{\log(z_{Ref}/z_0)}\right]^2$$

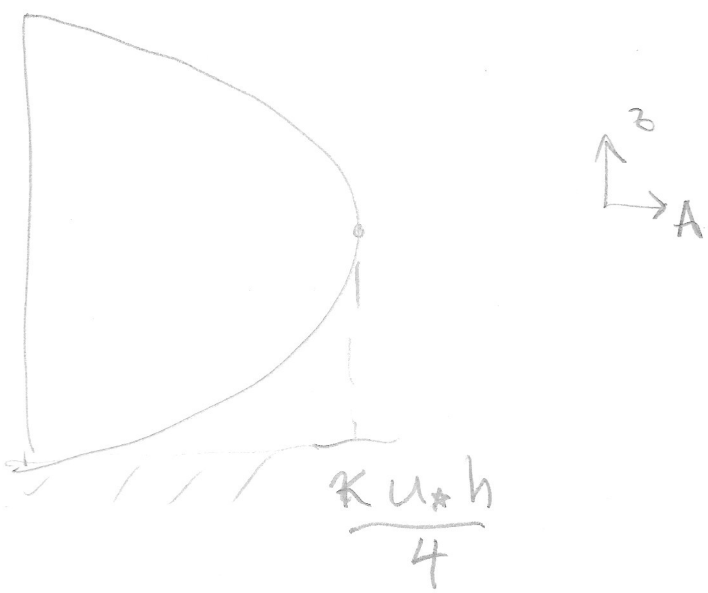
$\boxed{\text{Relating to Eddy Viscosity}}$

$$A \frac{u_*}{\kappa z} = \frac{u_*^2}{h}(h-z)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x} = \frac{1}{\rho_0}\frac{\partial \tau}{\partial z} = \frac{\partial}{\partial z}\left(A \frac{\partial u}{\partial z}\right) = \frac{\tau_B}{\rho_0 h} = \frac{u_*^2}{h}$$

$$u = \frac{u_*}{\kappa}\log\frac{z}{z_0} \Rightarrow \frac{\partial u}{\partial z} = \frac{u_*}{\kappa z}$$

$$\Rightarrow A = \kappa u_* z\left(1 - \frac{z}{h}\right) \quad \text{a parabola}$$



This gives rise to a parabolic velocity profile whereas what actually occurs is a log layer

Homework: compare for $\frac{1}{\rho_0}\frac{\partial p}{\partial x} = 10^{-4}\ m\,s^{-2}$

$C_{DR} = 3\times10^{-3}$, $h = 10\ m$

$z_{Ref} = 1\ m$